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A COMPARISON OF TWO TECHNIQUES FOR ESTIMATING THE TRAVEL
TIME OF AN ACOUSTIC WAVEFRONT BETWEEN TWO RECEIVING SENSORS

BY

FRANK J. MONTALBANO
B.S.E.E., Florida Atlantic University, 1979

RESEARCH REPORT

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in the Graduate Studies Program of the College of Engineering
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Spring Term
1984

ABSTRACT

In recent years the United States Navy has concentrated most of its Anti-Submarine Warfare (ASW) research and development efforts toward passive sonar. Its ability to locate enemy targets without being detected gives the passive sonar system a supreme strategic advantage over its active counterpart.

One aspect of passive sonar signal processing is the time delay estimation of an underwater acoustic wavefront. From this estimation the location and velocity of the radiating source (target) can then be determined. This report compares two popular methods of estimating time delay utilizing computer simulations of each: the cross correlator and the beamformer.

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CHAPTER I. INTRODUCTION

The purpose of a sonar system is to detect acoustic signals produced by underwater targets (usually submarines). The 'signal' being considered is strictly passive, as opposed to active. In passive sonar, the signal is generated by a target and received by the sonar's listening device. In active sonar, a 'ping' signal is generated by the sonar towards the target and returns to the sonar in the form of an echo (also called echo ranging). Passive has become the most desired of the two in a battle situation because it does not give away the sonar's presence or location. Therefore, the tactical advantage actually rests with the opponent who makes the least noise; he is least likely to be heard and, at the same time, the most likely to hear. Figure 1 illustrates a passive sonar (shown as a hydrophone and towed array listening device) receiving the propeller noise from a nearby submarine. The hydrophone (sonobuoy) is transmitting acoustic information back to the ship via RF radio.

When using passive sonar, the operator (or detection system) must be able to do the following [NAVMAT, 1968]:

1. Distinguish the sound emitted by the target from the usual background noise.
2. Distinguish between the various kinds of ship sounds with a view to possible identification of the type of vessel emitting

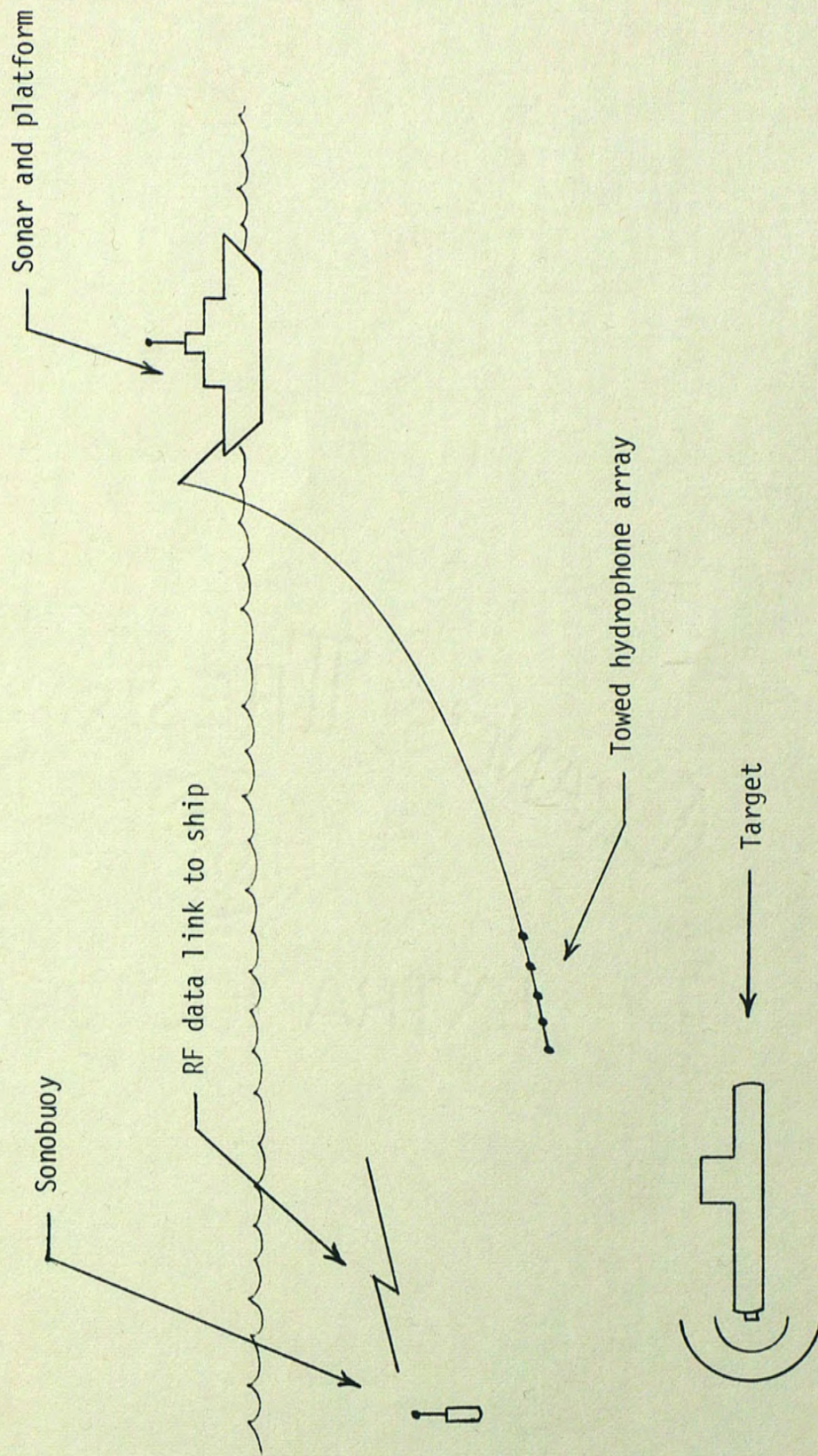


Figure 1. The Use of Passive Sonar

them and to obtain information on its operating conditions (such as the number of engines or generators running)

3. Having detected and perhaps partially identified a target, to obtain information concerning its approximate location and motion while it is still at a comparatively long range.

There are basically six factors to consider when receiving a passive signal:

1. Target Acoustics - The sound created by the vibration of a target's hull or propeller.
2. Transmission Loss - The 'weakening' of a target signal from the source to the receiver.
3. Ambient Background Noise - Generated by shipping traffic, biological life, and the sea itself.
4. Self Noise - Generated by the platform (or vessel) which carries the sonar.
5. Listening Equipment - The listening device (or hydrophone) used to transform acoustic energy into electrical energy.
6. Detection Threshold - The preassigned signal-to-noise ratio level of the decision as to target presence or absence.

In passive sonar, the objective is to use signals received by sensors in a phase-delayed manner so as to preferentially detect signals arriving from a particular direction (i.e., signals arriving on a particular beam) [Dudgeon, 1977]. This process is referred to as time delay estimation. After a target has been detected, the objectives of a sonar operator are to localize and track the target with respect to his own platform. In general terms, localization is the process of

determining the target range and bearing, while tracking is maintaining a history of previous target locations. For passive sonar systems, the target is detected from acoustic signals emitted by the target, and is localized using time delay estimations obtained from received signals at spatially separated hydrophones (acoustic sensors). Time Delay Estimation (TDE) is the process of estimating the travel time of an acoustic wavefront between two of these acoustic sensors. From the travel time, the relative position and velocity of the target, with respect to the sensors, can be determined. As shown in Figure 1, these acoustic sensors may be in the form of hydrophones along the length of a ship, a towed array of hydrophones, or as sonobuoys dropped in a set pattern by anti-submarine warfare aircraft.

Two common methods for estimating the time delay* of an acoustic wavefront between two receiving sensors are cross correlation and beamforming. This report will describe both methods of time delay estimation and compare their performance under simulated conditions.

* There exists a number of methods for estimating time delay. In fact, the U.S. Navy held a symposium in May of 1979 at the Naval Postgraduate School in Monterey, California where 100 prominent U.S. researchers participated in a Time Delay Estimation (TDE) Conference. From this conference, 29 papers were generated: seventeen papers on fixed TDE, five on variable TDE, and seven correspondence contributions.

CHAPTER II. TWO TECHNIQUES OF ESTIMATING TIME DELAY

In the ocean medium, the target-generated acoustic wave fronts arrive at each sensor through more than one path. This multipath effect is produced by signal reflections from the ocean surface and bottom causing the receiver to see the original signal plus attenuated and delayed versions of itself with additive uncorrelated noise (Figure 2). Since more sophisticated propagation modeling is required to simulate this complex effect, only the single, direct propagation path will be considered in this report. Also, it will be assumed that all of the targets, sensors, and propagation paths lie within the same plane. This two-dimensional simplification will allow the reader to better understand the problem of estimating time delay.

A simple mathematical model for the received target signals at the two sensors in the planar case is

$$x_1(t) = s(t) + n_1(t) \quad (1)$$

$$x_2(t) = \alpha s(t-D) + n_2(t) \quad (2)$$

where the source signal $s(t)$ and the background noises $n_1(t)$ at the first sensor, and $n_2(t)$ at the second sensor are Gaussian, stationary, and uncorrelated, D is the actual time delay between the sensors, and α is the signal attenuation. Once the signal has been detected, the

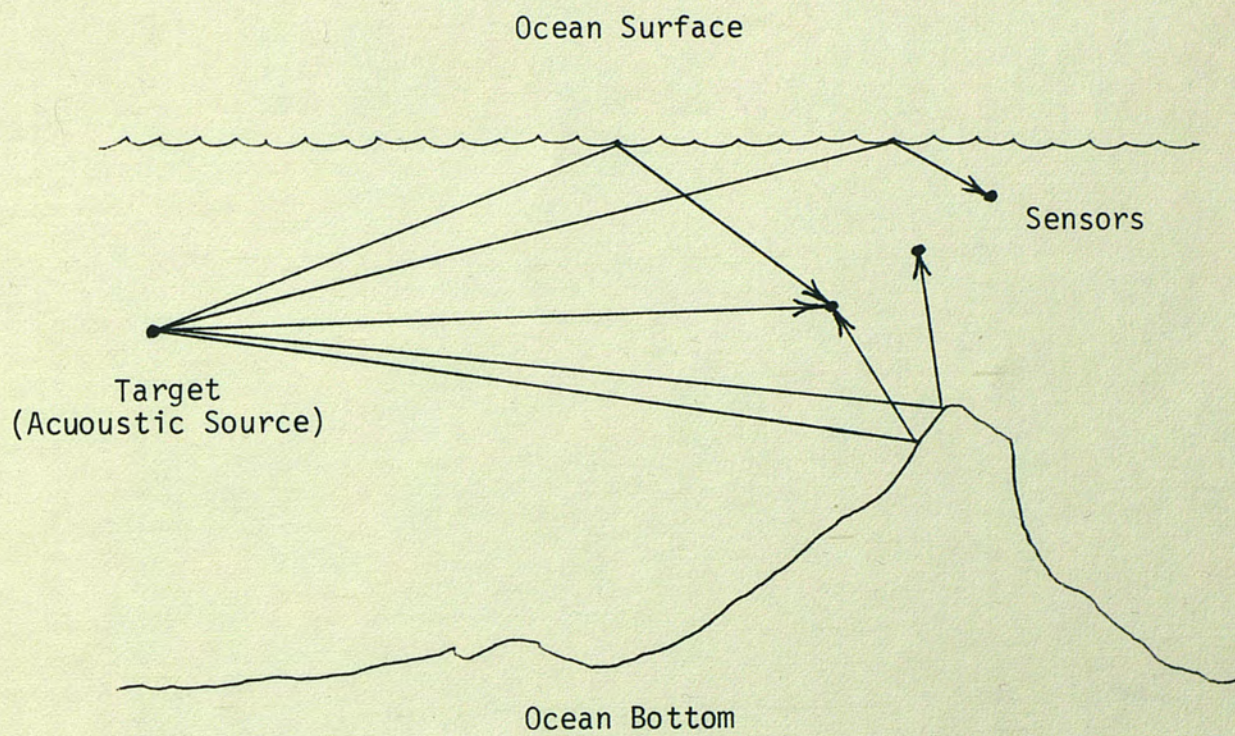


Figure 2. Multipath Effect

estimated time delay (\hat{D}) between the two sensors separated by the length L can be used to calculate the bearing angle given by [Carter, 1981]

$$\hat{B} \cong \sin^{-1} (c\hat{D}/L) \quad (3)$$

where

- 1) c is the speed of sound in water (independent of frequency)
- 2) \hat{B} is the bearing (angle) estimate
- 3) \hat{D} is the time delay estimate

As seen by Figure 3 and the bearing equation, the accuracy of the time delay estimate is critical to the accuracy of the target's bearing and range.

The following is a description of two common methods for estimating this time delay.

Cross Correlator

There are basically two methods of determining cross correlation $R_{x_1x_2}(\tau)$. One method computes the cross correlation by taking the expected value of the product of the first signal and the advanced second signal, as seen in Figure 4 [Knapp and Carter, 1979]. The other method computes the cross correlation by taking the inverse Fourier transform of the cross-power density spectrum ($S_{x_1x_2}(f)$) of the two received signals,

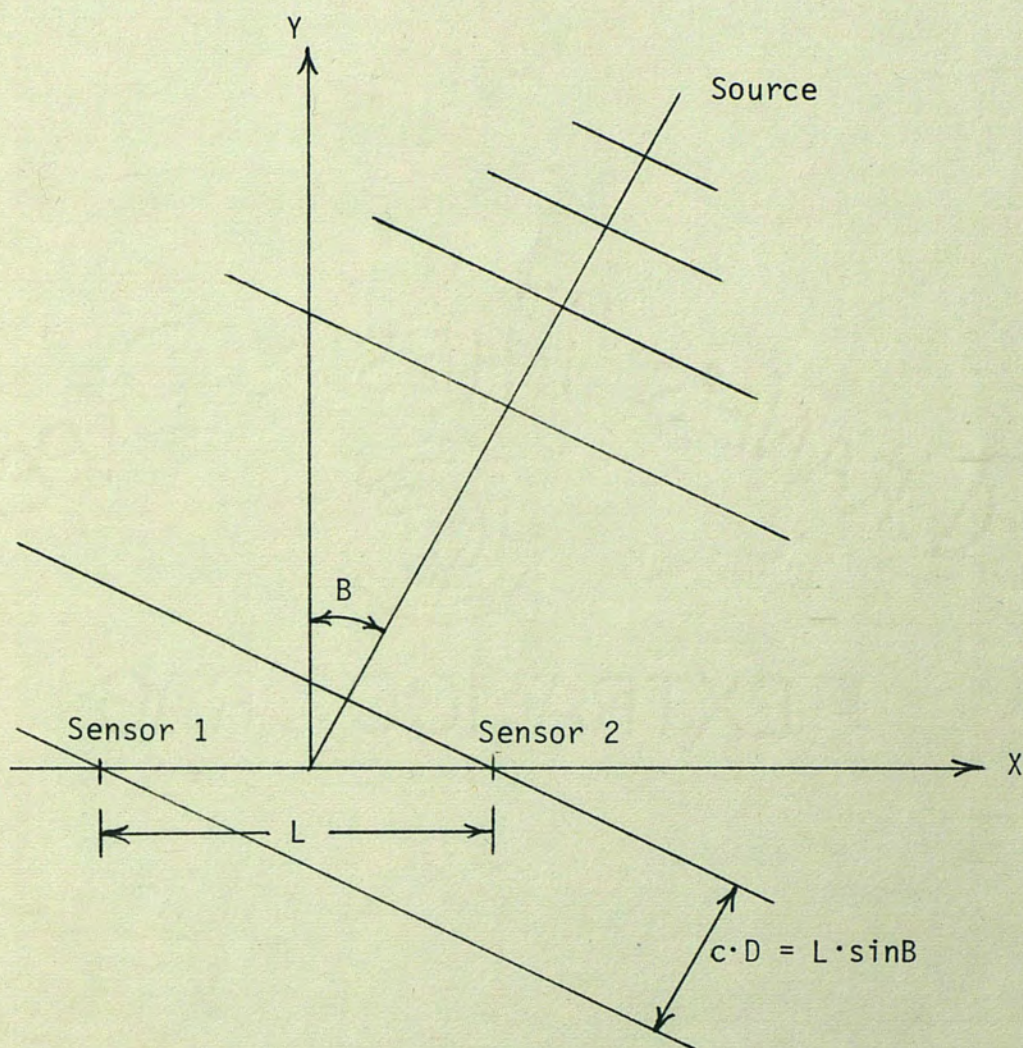


Figure 3. Acoustic Wave Fronts Approaching Two Sensors

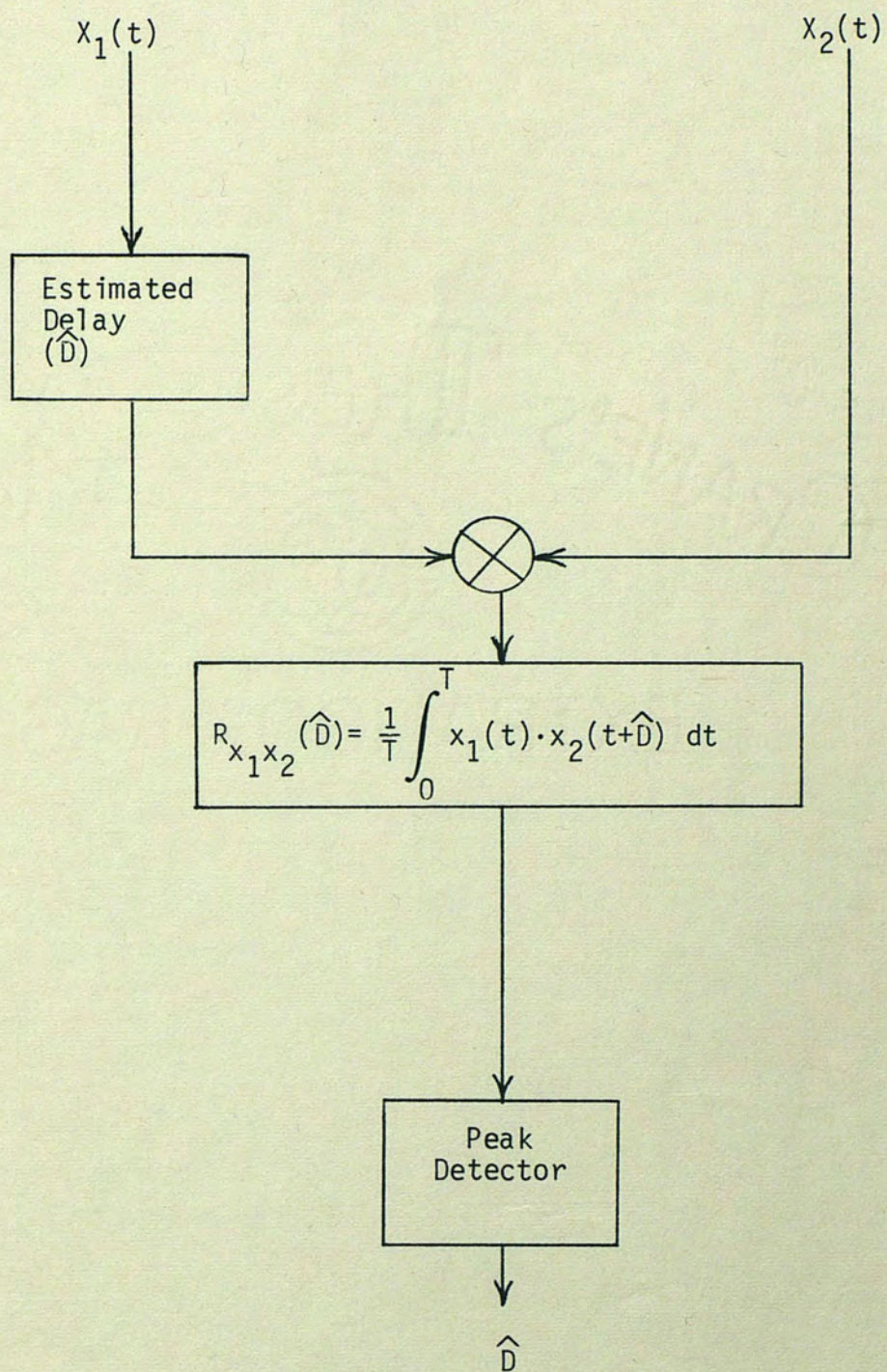


Figure 4. First Method of Cross Correlation

as seen in Figure 5 [Hassab and Boucher, 1979]. The relationship between the two methods is given by

$$\text{F.T. } [R_{x_1 x_2}(\tau)] = S_{x_1 x_2}(f). \quad (4)$$

The cross correlation of two real, periodic waveforms $x_1(t)$ and $x_2(t)$ is

$$R_{x_1 x_2}(\tau) = \frac{1}{T} \int_0^T x_1(t) x_2(t + \tau) dt. \quad (5)$$

Defining τ as the estimated delay (\hat{D}), equation (5) becomes

$$R_{x_1 x_2}(\hat{D}) = \frac{1}{T} \int_0^T x_1(t) x_2(t + \hat{D}) dt. \quad (6)$$

This equation is represented by the block diagram of Figure 4.

Substituting the received signal equations (1) and (2) for $x_1(t)$ and $x_2(t)$, equation (6) now becomes

$$R_{x_1 x_2}(\hat{D}) = \frac{1}{T} \int_0^T [s(t) + n_1(t)] [\alpha s(t - D + \hat{D}) + n_2(t + \hat{D})] dt. \quad (7)$$

After multiplication, equation (7) becomes

$$R_{x_1 x_2}(\hat{D}) = \frac{1}{T} \int_0^T [\alpha s(t) s(t - D + \hat{D}) + s(t) n_2(t + \hat{D}) + \alpha s(t - D + \hat{D}) n_1(t) + n_1(t) n_2(t + \hat{D})] dt. \quad (8)$$

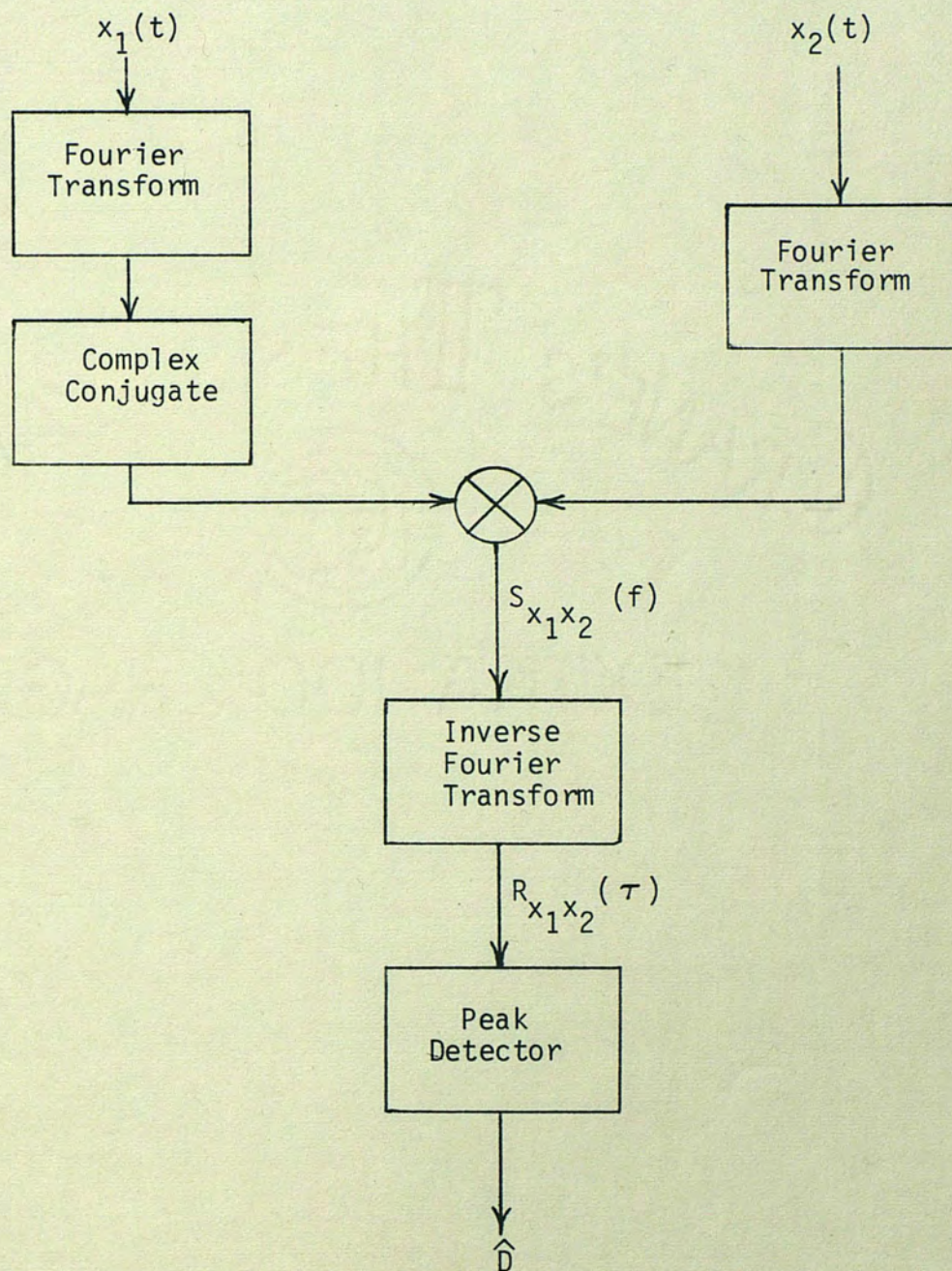


Figure 5. Second Method of Cross Correlation

Since $s(t)$ and $n(t)$ are uncorrelated, their cross correlation yields

$$\frac{1}{T} \int_0^T s(t) n_2(t+\hat{D}) dt = \frac{1}{T} \int_0^T \alpha s(t-D+\hat{D}) n_1(t) dt = 0 \quad (9)$$

and equation (8) now becomes

$$R_{x_1 x_2}(\hat{D}) = \frac{1}{T} \int_0^T [\alpha s(t) s(t-D+\hat{D}) + n_1(t) n_2(t+\hat{D})] dt \quad (10)$$

or

$$R_{x_1 x_2}(\hat{D}) = \alpha R_{ss}(\hat{D}-D) + R_{n_1 n_2}(\hat{D}). \quad (11)$$

Assuming the noise signals $n_1(t)$ and $n_2(t)$ are uncorrelated, equation (11) now becomes

$$R_{x_1 x_2}(\hat{D}) = R_{ss}(\hat{D}-D). \quad (12)$$

By definition, the autocorrelation function $R_{ss}(\hat{D}-D)$ should reach a maximum at $D=\hat{D}$. From equation (12), the cross correlation function $R_{x_1 x_2}(\hat{D})$ will also reach a maximum at $D=\hat{D}$. As seen by Figure 4, the output of the cross correlator is processed through a peak detector. This is designed to detect the maximum value of $R_{x_1 x_2}(\hat{D})$ at the estimated delay, which should correspond to the actual delay between the two received signals.

Beamformer

Because sonar systems of the past were not capable of processing large quantities of input data, they were usually limited to one or two sensors. But with the growth of high speed digital electronics, it has become more feasible to use computers and special purpose digital processors to perform the many computational tasks associated with the signal reception using a directional array. One such directional array is the beamformer. For the conventional beamforming of this report, the output of each hydrophone of the sensor array is time delayed and summed in the beamforming operation (Figure 6). Also, it will be assumed that consecutive sensors are spaced at equal distances from each other (linear array).

From Figure 7, the received signal can be modeled as a propagating plane wave which has a constant magnitude and phase along a line perpendicular to the direction of propagation.

The signal received at the i th sensor is given by

$$r_i(t) = s(t-D_i) + n_i(t) \quad (13)$$

where $s(t)$ is the target signal, $n_i(t)$ is the noise present at the i th sensors, and D_i is the time delay at the i th sensor.

The time delay between two consecutive sensors is

$$D = \frac{L \sin B}{c} \quad (14)$$

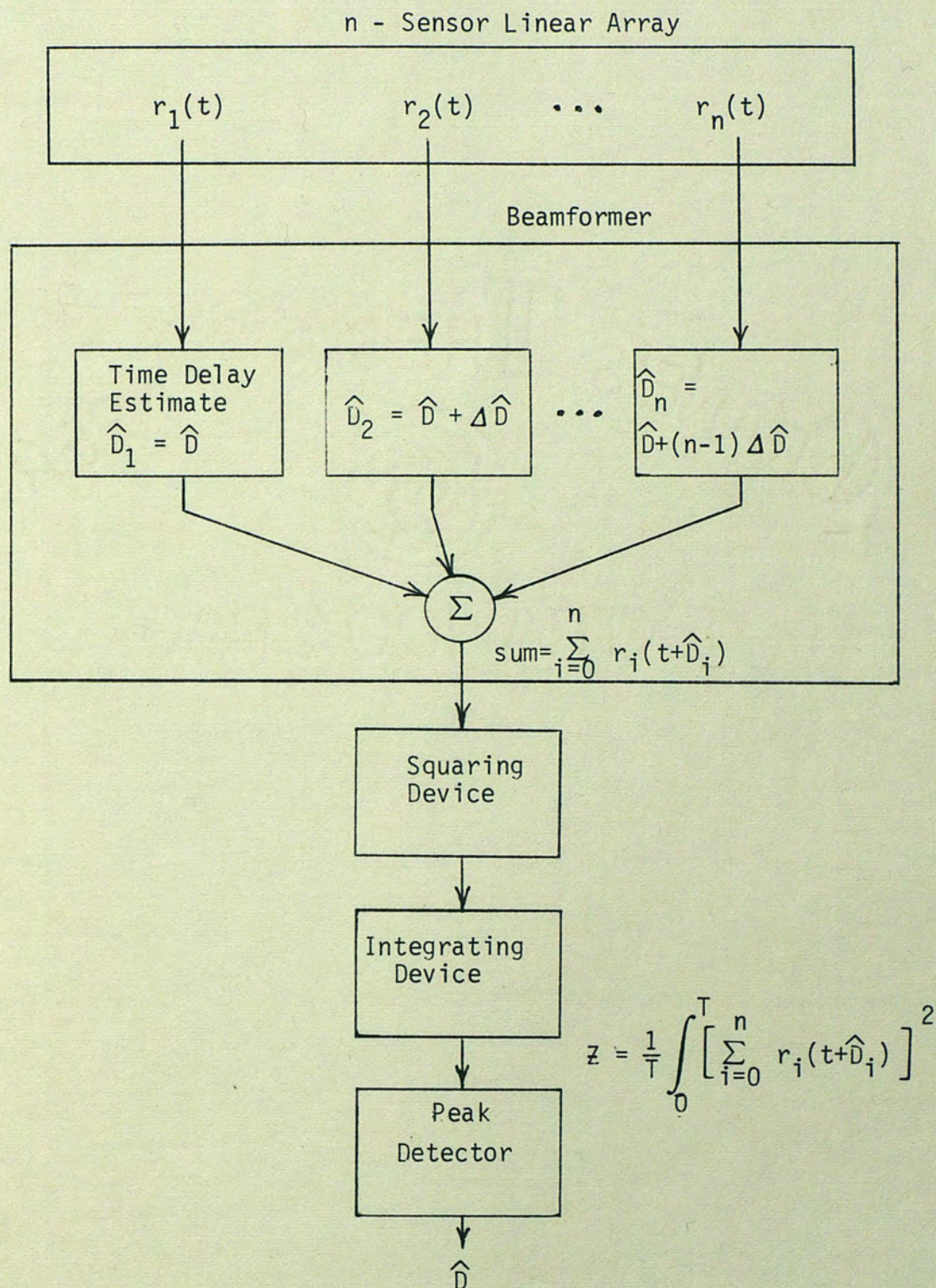


Figure 6. The Beamformer

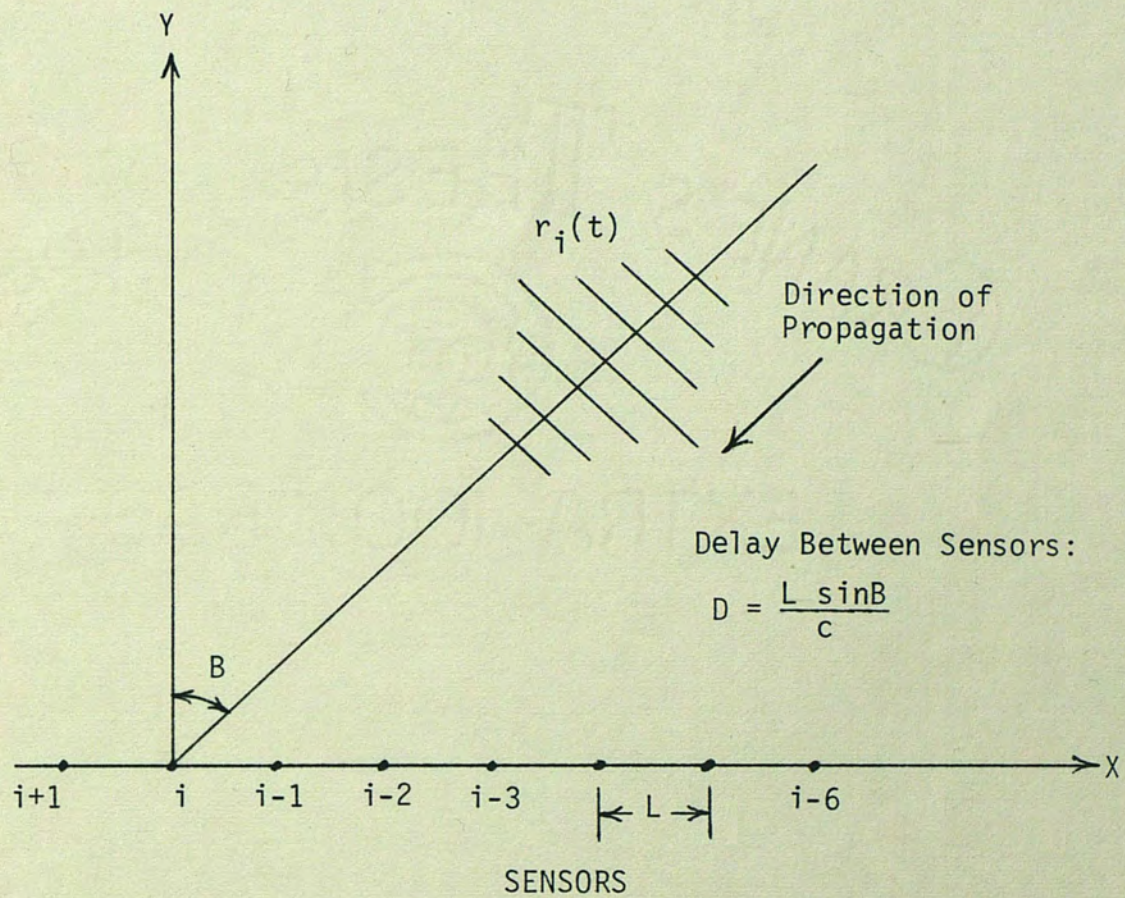


Figure 7. A Plane Wave Approaching a Sensor Array

where the wave is propagating at a velocity c in a direction at an angle B to the y -axis, and the sensors are spaced uniformly by a distance L .

If an estimated delay \hat{D} is added to the received signal at each sensor, the signal becomes

$$r_i(t + \hat{D}) = s(t - D_i + \hat{D}_i) + n_i(t + \hat{D}). \quad (15)$$

If these delayed signals are now summed, squared, and time averaged over one period of the signal, the output Z of the beamformer of Figure 6 becomes

$$Z = \frac{1}{T} \int_0^T \left[\sum_{i=0}^n r_i(t + \hat{D}) \right]^2 dt \quad (16)$$

where n is the total number of sensors (or received signals) in the array.

By expanding the product in equation (16), the output now becomes

$$Z = \frac{1}{T} \int_0^T [r_1^2(t + \hat{D}_1) + r_1(t + \hat{D}_1) r_2(t + \hat{D}_2) + \dots + r_n^2(t + \hat{D}_n)] dt \quad (17)$$

or

$$Z = \frac{1}{T} \int_0^T r_1^2(t + \hat{D}_1) dt + \frac{1}{T} \int_0^T r_1(t + \hat{D}_1) r_2(t + \hat{D}_2) dt + \dots + \frac{1}{T} \int_0^T r_n^2(t + \hat{D}_n) dt. \quad (18)$$

Since some of these integrals represent the basic definition of cross correlation (see equation (5)), equation (18) can be rewritten as

$$Z = \frac{1}{T} \int_0^T r_1^2(t + \hat{D}_1) dt + R_{r_1 r_2}(\hat{D}_2 - \hat{D}_1) + R_{r_1 r_3}(\hat{D}_3 - \hat{D}_1) + \dots \quad (19)$$

The remaining integrals become numerical values after integration because their values do not depend upon the estimated time delay \hat{D}_i . If we disregard the numerical values, the output of the beamformer becomes the sum of the cross correlation functions of all permutations of the n received signals taken two at a time.

The total number of permutations or cross correlation functions is defined as

$$P_{n, 2} = \frac{n!}{(n-2)!} \quad (20)$$

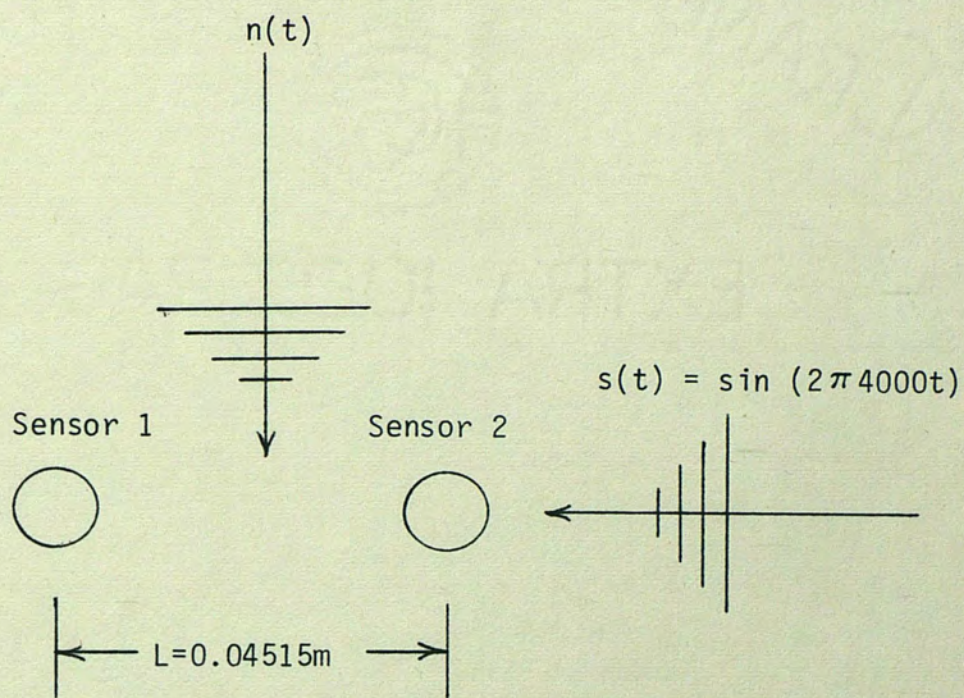
As defined previously for the single cross correlation, each cross correlation function for the beamformer should also reach a maximum at $D = \hat{D}$ if the noise signals are uncorrelated. A comparison of the accuracy of the cross correlator and the beamformer will be made in the sections to follow.

CHAPTER III. COMPUTER SIMULATION OF BOTH ALGORITHMS

In order to compare the cross correlator and beamformer, both methods of estimating time delay were simulated using a Hewlett-Packard HP-41C computer with a Math Pac module for integration. They were each tested for their accuracy in estimating the actual time delay of a sinusoidal signal propagating between two consecutive sensors while being disturbed by an additive noise source approaching from a different angle of inclination (see Figure 8). The most important consideration when simulating these algorithms is not the actual modeling of the signal and noise, but the fact that the same signal and noise models must be used for both methods. It is for this reason that simple models for the received signal and noise waveforms were used.

The actual conditions used for the simulation were as follows:

1. Signal: $s(t) = \sin(2\pi 4000t)$
2. Noise sources:
 $n_1(t) = 0.1 \cos(2\pi 1000t)$
 $n_2(t) = 0.1 \cos(2\pi 2000t)$
 $n_3(t) = 0.1 \cos(2\pi 3000t)$
 $n_4(t) = 0.1 \cos(2\pi 4000t)$
 $n_5(t) = 0.1 \cos(2\pi 5000t)$
 $n_6(t) = 0.1 \cos(2\pi 6000t)$
 $n_7(t) = 0.1 \cos(2\pi 7000t)$
3. Velocity of sound in sea water (3.6% salinity, 15°C): $c = 1,505$ meters/second



$$D_s = 30 \text{ microseconds}$$

$$D_n = 0 \text{ microseconds}$$

$$C = 1,505 \text{ m/sec}$$

Figure 8. Simulation Model

4. Actual signal angle of inclination (or bearing): $B = 90^0$ (relative to y-axis)
5. Noise source angle of inclination (or bearing): $B = 0^0$ (relative to y-axis)
6. Distance between sensors: $L = 0.04515$ meters
7. Actual time delay between sensors:

Signal delay:	$D_s = 30$ microseconds
Noise delay:	$D_n = 0$ microseconds

The actual delays were calculated from equation (3) and the values of the bearing, velocity of sound, and the distance between sensors given above.

Only one noise source ($n_1(t)$ through $n_7(t)$) was used for each estimation of time delay. Thus, seven separate estimations were performed with seven different noise sources present using both algorithms. For example, the output of the cross correlator from equation (7), assuming no attenuation between sensors, using noise source $n_1(t)$ is

$$R_{x_1 x_2}(\hat{D}) = 4000 \int_0^{\frac{1}{4000}} [\sin(2\pi 4000t) + 0.1 \cos(2\pi 1000t)]$$

$$[\sin(2\pi 4000(t - D_s + \hat{D})) + 0.1 \cos(2\pi 1000(t - D_n + \hat{D}))] dt$$

where

$$T = \frac{1}{4000} = 250 \text{ microseconds}$$

(21)

and the output of the beamformer from equation (16) with the same noise source is

$$z = \int_0^{\frac{1}{4000}} \left[\sum_{i=0}^3 [\sin [2\pi 4000(t - D_{si} + \hat{D}_i)] + \cos [2\pi 1000(t - D_{ni} + \hat{D}_i)]]^2 dt. \quad (22)$$

Appendix A describes the simulation program for the cross correlator (equation 21) and Appendix B describes the simulation program for the beamformer (equation 22). Equation (22) assumes an array of four sensors.

Figures 9 and 10 depict the output of each method assuming there is no noise present in order to show how each of the simulated methods estimates the actual time delay of 30 microseconds. The maximum value of the output along the y-axis corresponds to the estimated delay (\hat{D}) along the x-axis. In the cases of Figures 9 and 10, the estimated delay is equal to the actual delay of the signal since there is no noise present. As each noise source is added to the target signal at the input, the estimated delay begins to vary. The estimated time delay values and errors for both methods, given each of the seven different noise sources, are shown in Figure 11. These values were calculated from the simulation programs of Appendices A and B.

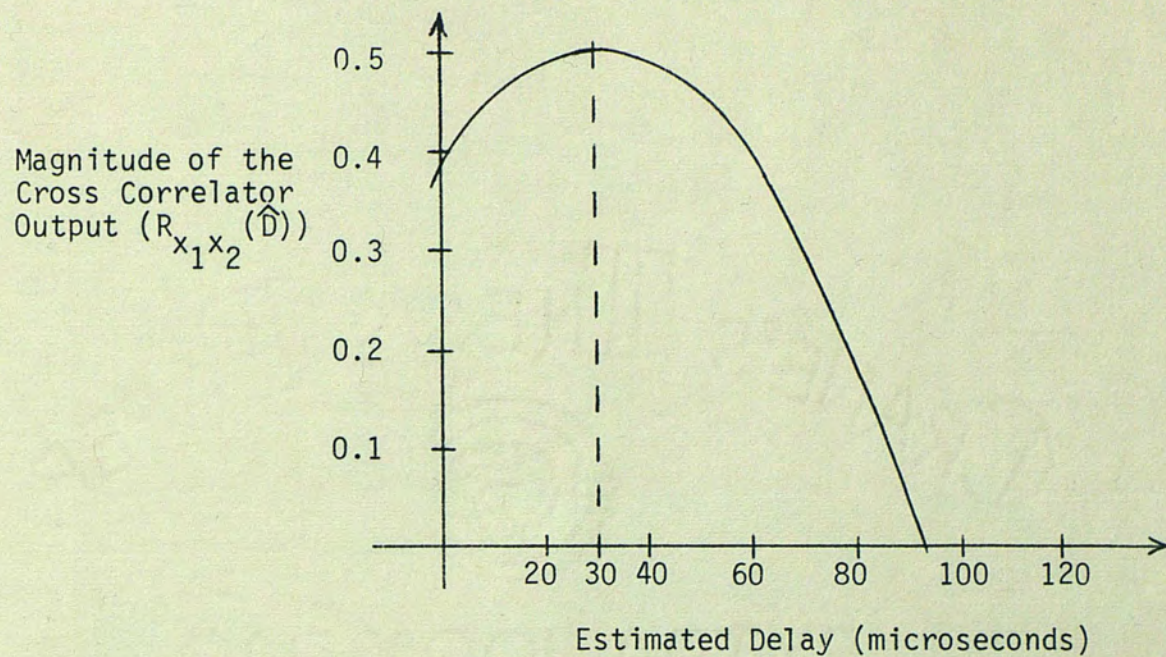


Figure 9. Cross Correlator Simulation Output [$n(t)=0$]

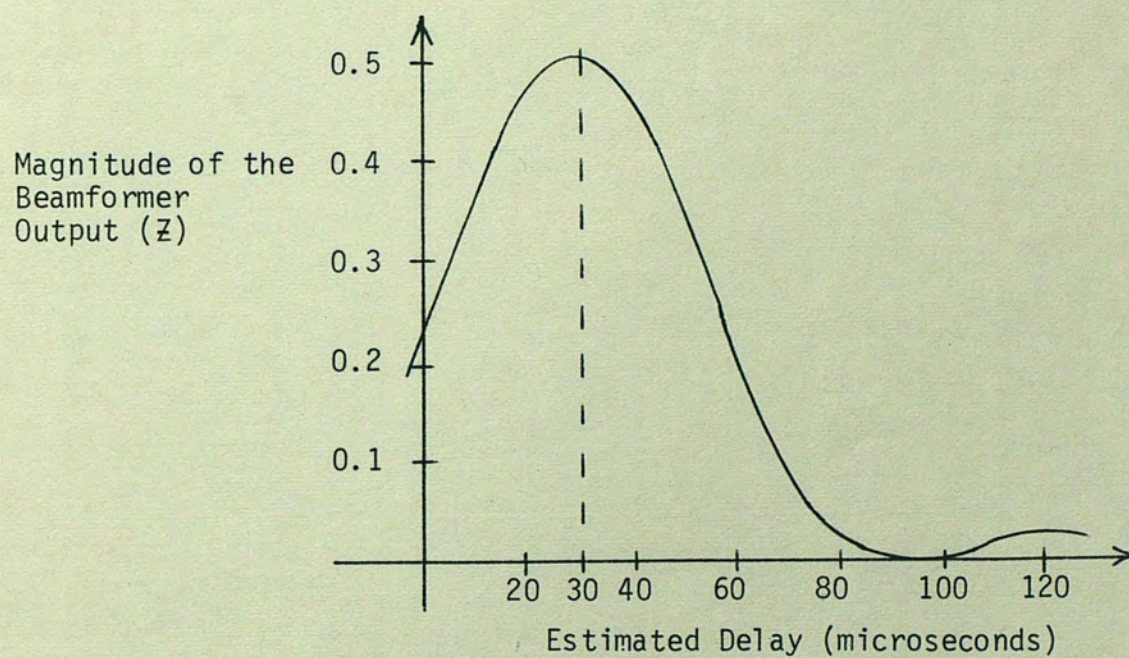


Figure 10. Beamformer Simulation Output [$n(t)=0$]

<u>Noise</u> <u>Source</u>	<u>Cross Correlator</u>			<u>Beamformer</u>		
	<u>Estimated</u>			<u>Estimated</u>		
	<u>Delay</u> (micro-seconds)	<u>Error</u> (micro-seconds)	<u>Percent</u> <u>Error</u>	<u>Delay</u> (micro-seconds)	<u>Error</u> (micro-seconds)	<u>Percent</u> <u>Error</u>
$n(t) = 0$	30.0	0.0	0.0%	30.0	0.0	0.0%
$n_1(t)$	29.9	0.1	0.3%	29.9	0.1	0.3%
$n_2(t)$	29.8	0.2	0.6%	29.5	0.5	1.7%
$n_3(t)$	29.0	1.0	3.3%	30.0	0.0	0.0%
$n_4(t)$	31.0	-1.0	3.3%	32.2	2.2	7.3%
$n_5(t)$	33.7	-3.7	12.3%	31.8	1.8	6.0%
$n_6(t)$	31.8	-1.8	6.0%	28.9	1.1	3.7%
$n_7(t)$	27.6	2.4	8.0%	29.7	0.3	1.0%

Figure 11. Estimated Time Delay and Errors

CHAPTER IV. COMPARISONS AND CONCLUSION

A number of comparisons can be drawn from the results of Figure 11. As one would expect, the time delay estimate error is greater for both techniques as the noise source frequency approaches the signal frequency of 4000 Hertz. However, the cross correlator seems to behave more erratically than the beamformer, as seen by the larger errors between the actual and estimated time delay values.

The errors of the estimated delay from the true delay of 30 microseconds for the cross correlator and beamformer are given in Figure 11.

The mean time delay estimate for the cross correlator is

$$m = \frac{\sum X_i}{N} = 30.40 \quad (23)$$

while the mean time delay estimate for the beamformer is

$$m = 30.28 \quad (\text{actual delay} = 30 \text{ microseconds}).$$

The mean square difference (or error) between the estimated delays and the actual delay of 30 microseconds for the cross correlator is

$$E [(\hat{D} - D)^2] = 3.53 \quad (24)$$

while the mean square difference for the beamformer is

$$E[(\hat{D} - D)^2] = 1.38.$$

From these comparisons, it can be concluded that the beamformer outperforms the cross correlator for the simulated conditions given. Figures 9 through 12 show the beamformer is not only more accurate in estimating time delay, but the detection peaks are sharper and less erratic than those of the correlator.

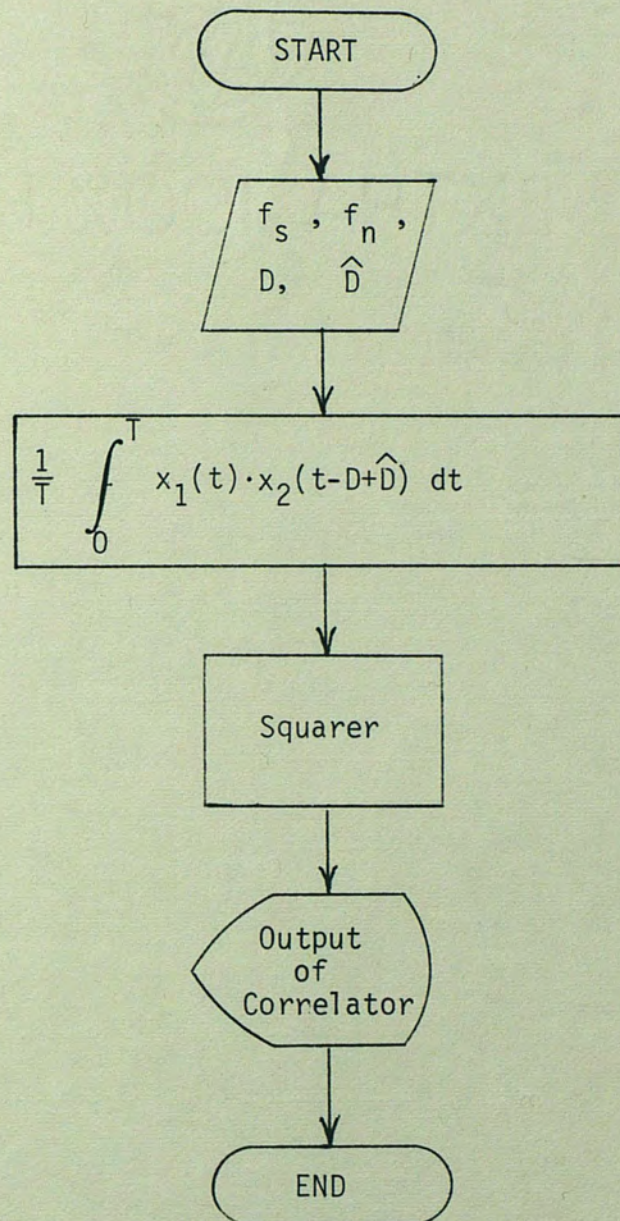
It can also be shown that the beamformer is more accurate in estimating time delay when using a greater number of sensors. For instance, when we corrupt the target signal with noise source $n_4(t)$, the beamformer estimates the time delay to be 32.2 microseconds (7.3% error) using four sensors in the array. If we double the number of sensors to eight, the estimated time delay using the same noise source is 30.4 microseconds (1.3% error). Obviously, this is a hardware cost vs. beamformer performance factor that one must consider when evaluating how many sensors are required. Other system parameters such as the number of beams, cable bandwidth, digital memory, reliability, and maintainability should also be considered when designing a passive target detection system.

APPENDIX A

HP SIMULATION PROGRAM FOR CROSS CORRELATION

Note: The simulation program contained in this appendix is for use on the Hewlett-Packard Model HP-41C or HP-41CV programmable calculator with the Hewlett-Packard Math Pac module for integration. The program is designed to ask the operator for all of the required inputs. The frequencies should be in Hertz and the time delays in microseconds.

FLOW CHART



Listing

<u>Code</u>	<u>Comments</u>
01 LBL "Cross Correlate"	Program Title
02 "Freq of Signal?"	
03 PROMPT	
04 2	
05 X	
06 π	
07 X	
08 STO 99	$2 \pi f_s$
09 "FREQ of NOISE?"	
10 PROMPT	
11 2	
12 X	
13 π	
14 X	
15 STO 98	$2 \pi f_n$
16 "Actual Delay?"	
17 Prompt	
18 STO 97	D
19 "Estimated Delay?"	
20 Prompt	
21 STO 96	\hat{D}
22 "Integrate"	

23 AVIEW

24 PSE

25 INTG

For integration of function
name, specify: 'X1X2'

26 RCL 99

27 X

28 X 2

29 STO 95

$$\left[\frac{1}{T} \int_0^T x_1(t) x_2(t-D+\hat{D}) dt \right]^2$$

30 "CC Output ="

31 ARCL 95

32 AVIEW

33 END

01 LBL X1X2

02 STO 94

t

03 RCL 96

\hat{D}

04 +

05 STO 93

(t + \hat{D})

06 RCL 97

D

07 -

08 STO 92

(t - D + \hat{D})

09 RCL 99

$2\pi f_s$

10 X

11 SIN

$\sin [2\pi f_s(t-D+\hat{D})]$

12 RCL 98

13 RCL 93

14 X
 15 COS
 16 +
 17 RCL 99

$$\cos [2 \pi f_n (t + \hat{D})]$$

18 RCL 94

19 X

20 SIN

$$\sin [2 \pi f_s t]$$

21 RCL 98

22 RCL 94

23 X

24 COS

$$\cos [2 \pi f_n t]$$

25 +

26 X

$$[\sin (2 \pi f_s t) + \cos (2 \pi f_n t)]$$

27 RTN

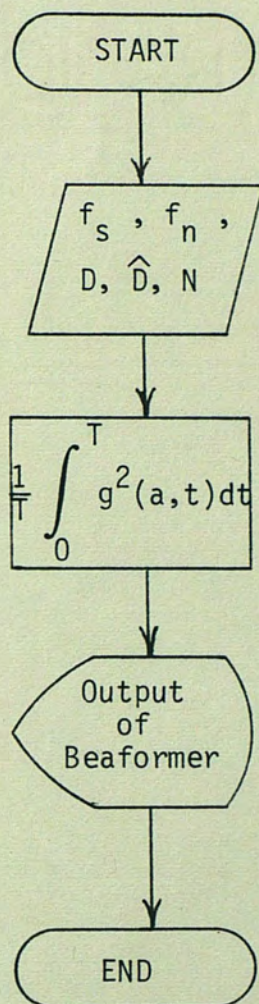
$$[\sin (2 \pi f_s (t - D + \hat{D})) + \cos (2 \pi f_n (t + \hat{D}))]$$

APPENDIX B

HP SIMULATION PROGRAM FOR BEAMFORMING

Note: The simulation program contained in this appendix is for use on the Hewlett-Packard Model HP-41C or HP-41CV programmable calculator with the Hewlett-Packard Math Pac module for integration. The program is designed to ask the operator for all of the required inputs. The frequencies should be in Hertz and the time delays in microseconds.

FLOW CHART



LISTING

<u>CODE</u>		<u>COMMENTS</u>
01	LBL "BEAMFORM"	PROGRAM TITLE
02	"FREQ OF SIGNAL?"	
03	PROMPT	
04	2	
05	X	
06	π	
07	X	
08	STO 99	$2 \pi f_s$
09	"FREQ OF NOISE?"	
10	PROMPT	
11	2	
12	X	
13	π	
14	X	
15	STO 98	$2 \pi f_n$
16	"ACTUAL DELAY?"	
17	PROMPT	
18	STO 97	D
19	"ESTIMATED DELAY?"	
20	PROMPT	
21	STO 96	\hat{D}
22	"NUMBER OF SENSORS?"	

23 PROMPT
 24 STO 95
 25 "INTEGRATE"
 26 AVIEW
 27 PSE
 28 INTG

For integration of function
 name, specify "G"

29 RCL 99
 30 X
 31 STO 94
 32 "BF OUTPUT ="
 33 ARCL 94
 34 AVIEW
 35 END

$$\frac{1}{T} \int_0^T \left[\frac{1}{N} \sum_{i=0}^{N-1} \sin [2 \pi f_s (t - iD + i\hat{D})] + \cos [2 \pi f_n (t + i\hat{D})] \right]^2 dt$$

01 LBL G
 02 STO 93
 03 0
 04 STO 92
 05 RCL 95
 06 STO 91
 07 LBL 01
 08 RCL 96
 09 RCL 97
 10 -
 11 RCL 91

t

N

N = i

START LOOP

\hat{D}

D

N

12	1	
13	-	
14	X	$i (\hat{D}-D)$
15	RCL 93	t
16	+	
17	RCL 99	$2 \pi f_s$
18	X	
19	SIN	$\text{SIN} [2 \pi f_s (t-iD+i\hat{D})]$
20	RCL 91	
21	1	
22	-	
23	RCL 96	
24	X	
25	RCL 93	
26	+	
27	RCL 98	
28	X	
29	COS	$\text{COS} [2 \pi f_n (t+i\hat{D})]$
30	+	
31	RCL 92	$\sum_{i=0}^{N-1} \sin [2 \pi f_s (t-iD+i\hat{D})] +$
32	+	$\cos [2 \pi f_n (t+i\hat{D})]$
33	STO 92	
34	DSE 91	
35	GTO 01	END LOOP
36	RCL 92	

37 RCL 95

38 \div 39 x^2

40 RTN

$$\left[\frac{1}{N} \sum_{i=0}^{N-1} \sin [2 \pi f_s (t - iD + \hat{D})] + \cos [2 \pi f_n (t + i\hat{D})] \right]^2$$

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